AP CALCULUS SUMMER ASSIGNMENT

Hello Future AP Calculus Students!!!

The following is a sampling of precalculus topics that will be used throughout the course. Students strong in the knowledge of these topics typically have an easier time throughout the course. Because there will be no time to review in the fall, it is up to you to assess your own understanding of these before returning in August. To be successful in this course, you must have a strong intuitive understanding of these topics and be able to apply these skills to the many new concepts you will be learning. You will find that AP Calculus is unlike any math class you have taken before. You will need every math concept you have learned up to now – forgetting is not an option or an excuse acceptable in this course. Every concept will also build on the previous one, so you cannot let yourself ever fall behind.

The next few pages consist of an overview of the predominant precalculus topics (not all of course, but some of the most important!), a brief reminder of how to approach various topics, and some problems to help you assess your understanding of these topics. None of this material should be unfamiliar to you, but it needs to be fresh in your mind before returning in the fall. I especially want to stress that you should have key concepts like the unit circle and trigonometric identities MEMORIZED!!

***This assignment will be collected the first day of class and there will be a QUIZ on this material on day 3 ***

SAMPLE:

Trigonometry: Everything will be done in radians

Unit Circle!! You MUST know the 6 trig values of the unit circle very well. .

Ex. Sin $(2\pi/3)$ tan $(7\pi/6)$ cos $(7\pi/4)$ sin $(\pi/2)$

Graphs of all 6 trig functions - including domain, range, and period.

Identities: Pythagorean, double angle, sum and difference, even/odd

Solving Equations with trig (will need identities and unit circle)

Ex. $\text{Cos } 2x + \sin^2 x = 0$ (all real x) $2\sin^2 x = 3\sin x - 1 \quad 0 \leq x < 2\pi$

Graphical Analysis

Determine domain and range of function

Ex. $Y = \sqrt{6-x}$ $y = \dfrac{(x-3)(x+6)}{x^2-5x+6}$ $y = \log(3x-4)$

Translations and scale changes

Ex. Given the graph of f(x), be able to graph $f(x+2)$, $3f(x)$, $f(-x)$, $f(|x|)$, etc

Graphs of "toolkit" functions including all domain and range

Ex. $Y = x^2$ $y = x^3$ $y = \sqrt{x}$ $y = \ln x$ $y = 3^x$ $y = \dfrac{1}{x}$

Solving Equations and inequalities

Finding inverses of functions (including trig)

Logarithms and exponentials: MUST know properties and how to use in solving equations

Ex. Solve: $2\ln(x-2) - \ln(x) = \ln(2)$
Solve: $e^{3\ln x - \ln 2} = 4$

Test Point Method to solve polynomial and rational inequalities. Express solutions in interval notation.

Ex. Solve: $x^2(x-6)(x^2-9) > 0$ Solve: $\dfrac{(x+2)(x-1)^3}{(x+4)^2} < 0$

Solving ALL types of equations – log, exponential, trig, rational, polynomial, radical, etc.

Solutions: (to examples pg. 1)

$\sin(2\pi/3) = \frac{\sqrt{3}}{2}$ $\tan(7\pi/6) = \frac{\sqrt{3}}{3}$ $\cos(7\pi/4) = \frac{\sqrt{2}}{2}$ $\sin(\pi/2) = 1$

$\sin(2x) = 2\sin x\cos x$
$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

Solve:

$\cos 2x + \sin^2 x = 0$ (all real x)
$1 - 2\sin^2 x + \sin^2 x = 0$
$1 - \sin^2 x = 0$ $\sin x = \pm 1$
$\sin^2 x = 1$ $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

$2\sin^2 x = 3\sin x - 1 \quad 0 \le x < 2\pi$
$2\sin^2 x - 3\sin x + 1 = 0$
$(2\sin x - 1)(\sin x - 1) = 0$
$\sin x = \frac{1}{2}$ $\sin x = 1$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{2}$

Determine the domain:

$Y = \sqrt{6-x}$
$x \le 6$
$(-\infty, 6]$

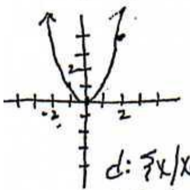
$y = \frac{(x-3)(x+6)}{x^2-5x+6}$
$(x-3)(x-2)$
$\{x \in \mathbb{R} \mid x \ne 3 \ \ x \ne 2\}$
$(-\infty,2) \cup (2,3) \cup (3,\infty)$

$y = \log(3x-4)$
$x > \frac{4}{3}$
$(\frac{4}{3}, \infty)$

Given the graph of f(x), be able to graph f(x+2), 3f(x), f(-x), f(|x|), etc
f(x+2) moves left 2. 3f(x): mult y's by 3 f(-x): reflects over y-axis
f(|x|): y values of neg. x's = y values of pos. x's

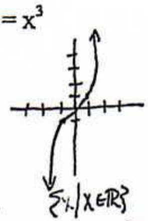
Graph and give domain and range.

$Y = x^2$



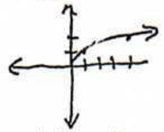
d: $\{x \mid x \in \mathbb{R}\}$
r: $\{y \mid y \ge 0\}$

$y = x^3$



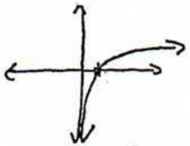
$\{x \mid x \in \mathbb{R}\}$
$\{y \mid y \in \mathbb{R}\}$

$y = \sqrt{x}$



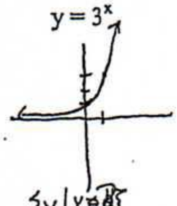
$\{x \mid x \ge 0\}$
$\{y \mid y \ge 0\}$

$y = \ln x$



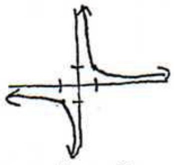
$\{x \mid x > 0\}$
$\{y \mid y \in \mathbb{R}\}$

$y = 3^x$



$\{x \mid x \in \mathbb{R}\}$
$\{y \mid y > 0\}$

$y = \frac{1}{x}$

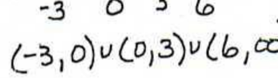


$\{x \mid x \ne 0\}$
$\{y \mid y \ne 0\}$

Solve: $2\ln(x-2) - \ln(x) = \ln(2)$
$\ln\frac{(x-2)^2}{x} = \ln 2$
$\frac{(x-2)^2}{x} = 2$
$(x-2)^2 = 2x$
$x^2 - 4x + 4 = 2x$
$x^2 - 6x + 4 = 0$
$x = \frac{6 \pm \sqrt{20}}{2}$
$x = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$
* $3-\sqrt{5}$ not in domain of $\ln x$
so $\boxed{x = 3+\sqrt{5}}$

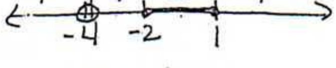
Solve: $e^{3\ln x - \ln 2} = 4$
$\cancel{e}^{\cancel{\ln}\frac{x^3}{2}} = 4$
$\frac{x^3}{2} = 4$ $x^3 = 8$ $\boxed{x = 2}$

Solve: $x^2(x-6)(x^2-9) > 0$



$(-3,0) \cup (0,3) \cup (6,\infty)$

Solve: $\frac{(x+2)(x-1)^3}{(x+4)^2} \le 0$



$[-2, 1]$

Notes that *might* help ☺

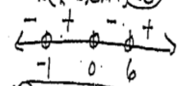
Solving Polynomial & Rational Inequalities

- Set inequality to zero.
- Put zeros of numerator & denominator on # line
- check values within each interval to see if positive or negative
- choose intervals want ⇒ write in interval notation

Ex. $x^3-5x^2<6x$

$x^3-5x^2-6x<0$

$x(x-6)(x+1)\boxed{<0}$ ⇒ want neg.



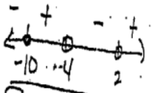
$\boxed{(-\infty,-1)\cup(0,6)}$

Ex. $\frac{2}{x-2}\le\frac{1}{x+4}$

$\frac{2}{x-2}-\frac{1}{x+4}\le 0$

$\frac{x+10}{(x-2)(x+4)}\le 0$

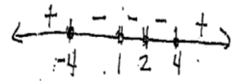
zeros of denom are *never* included in interval



$\boxed{(-\infty,-10]\cup(-4,2)}$

* signs will alternate unless have even roots

Ex. $(x-2)^2(x+4)(x-4)^3(x-1)^4\ge 0$ (I always put 2 lines ∥ when even root)



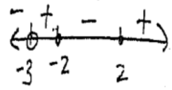
$\boxed{(-\infty,-4]\cup[4,\infty)}$

* If doesn't factor, graph on calculator to find zeros.

* Used in solving *any* polynomial or rational ineq. Sometimes need with domain...

Ex. Determine the domain of $y=\sqrt{\frac{x^2-4}{x+3}}$.

- Remember $\frac{x^2-4}{x+3}\ge 0$ & $x+3\ne 0$ to be in domain:



want +'s ⇒ $\boxed{(-3,-2]\cup[2,\infty)}$

GRAPHS $y=f(x)$

$y=f(x+c)$ ⇒ moves left c

$y=f(x-c)$ ⇒ moves rt c

$y=f(x)+c$ ⇒ moves up c

$y=f(x)-c$ ⇒ *down* c

$y=kf(x)$ ⇒ mult. *y's* by k

$y=f(kx)$ ⇒ mult. x's by $\frac{1}{k}$

$y=-f(x)$ ⇒ reflects over x-axis

$y=f(-x)$ ⇒ reflects over y-axis.

$y=|f(x)|$ ⇒ all y values are positive

$y=f(|x|)$ ⇒ all neg. x values have same y values as pos. x values.

Inverses

To find $f^{-1}(x)$, switch x & y & solve for y.

Remember, domain of $f(x)$ = range of $f^{-1}(x)$ & vice versa

Ex. Find the inverse of $f(x)=\sqrt[3]{x-2}+6$.

$x=\sqrt[3]{y-2}+6$

$x-6=\sqrt[3]{y-2}$

$(x-6)^3=y-2$

$\boxed{y=(x-6)^3+2}$

Ex. $f(x)=\frac{2x+1}{x-2}$. Find $f^{-1}(x)$

$x=\frac{2y+1}{y-2}$

$xy-2x=2y+1$

$xy-2y=2x+1$

$y(x-2)=2x+1$

$\boxed{y=\frac{2x+1}{x-2}}$ so $f^{-1}(x)=f(x)$ in this case.

Use inverses to solve equations! Remember logs & exponentials are inverses.

Ex. Solve $3^{x-2}=6$.

$\ln 3^{x-2}=\ln 6$

$(x-2)\ln 3=\ln 6$

$\boxed{x=\frac{\ln 6}{\ln 3}+2}$

Ex. Solve $\log_6(x+1)+\log_6 x=1$

$\log_6[x(x+1)]=1$

$x(x+1)=6^1$

$x^2+x=6$

$x^2+x-6=0$ $(x+3)(x-2)=0$

$\cancel{x=-3}$ $\boxed{x=2}$

Remember to chk ans. in *original* eqn. Can't take log of neg!

Ex. Solve $6e^{2x}=4$.

$e^{2x}=\frac{2}{3}$ ← Get e by itself *before* log.

$\ln(e^{2x})=\ln\frac{2}{3}$

$2x=\ln 2/3$

$\boxed{x=\frac{\ln 2/3}{2}}$

Ex. $3^{4\log_3 x-\log_3 4}=2$

$\cancel{3^{\log_3}}\frac{x^4}{4}=2$ ← Use properties of logs to combine exp.

$\frac{x^4}{4}=2$ ← Exp. base 3 & $\log_3$ cancel b/c inverses

$x^4=8$ $\boxed{x=\sqrt[4]{8}}$

TRIG EQS.

* *MUST* know unit circle values *extremely* well.
* Know identities!!

Ex. $2\sin^2 x-1=0$ $0\le x\le 2\pi$

$\sin^2 x=\frac{1}{2}$

$\sin x=\pm\sqrt{\frac{1}{2}}=\pm\frac{\sqrt{2}}{2}$

$\boxed{x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}}$

Ex. Solve $2\sin^2\theta+\sin 2\theta=0$ $\theta\in\mathbb{R}$

$2\sin^2\theta+2\sin\theta\cos\theta=0$

$2\sin\theta(\sin\theta+\cos\theta)=0$

$\sin\theta=0$ $\sin\theta=-\cos\theta$

$\boxed{\theta=k\pi \quad \theta=\frac{3\pi}{4}+2k\pi,\ \frac{7\pi}{4}+2k\pi \quad k\in\mathbb{Z}}$